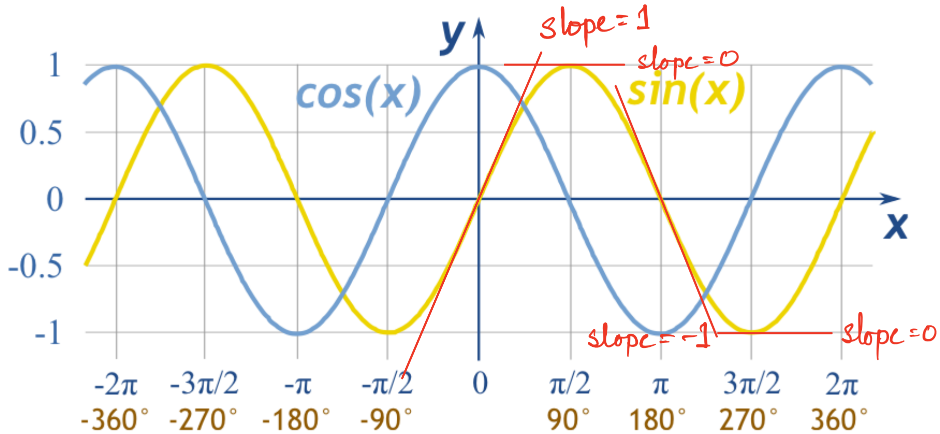
Derivative of Trigonometric Functions:



If we find the graph of the derivative of $\sin x$, then that resembles $\cos x$.

So, we get our first trigonometric derivative function

$$\frac{d}{dx}[\sin x] = \cos x.$$

Similarly we will get $\frac{d}{dx}[\cos x] = -\sin x$

Using these two we can derive $\frac{d}{dx}[\tan x]$.

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$

$$= \frac{\cos(x)\frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}(\cos x)}{(\cos x)^2}$$

$$= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{(\cos x)^2} = \frac{1}{\cos^2 x} = \sec^2 x.$$

Similarly, we can get the derivative of $\cot x$, $\sec x$, $\csc x$.

① $(\sin x)' = \cos x$

⊛ ② $(\cos x)' = -\sin x$

③ $(\tan x)' = \sec^2 x$

⊛ ④ $(\cot x)' = -\csc^2 x$

⑤ $(\sec x)' = \sec x \tan x$

⊛ ⑥ $(\csc x)' = -\csc x \cot x$

Loose Observation:

when trigonometric function starts with 'c', the result has a '-ve' sign.

① Find the equation of the tangent of $f(x) = 6\sin x$ at $x = \frac{\pi}{6}$

⊛ Need a point on the curve $f(x)$

at $x = \frac{\pi}{6}$, we have $f\left(\frac{\pi}{6}\right) = 6\sin\frac{\pi}{6} = 6 \cdot \frac{1}{2} = 3$

So, $\left(\frac{\pi}{6}, 3\right)$ lies on the curve $f(x)$ & is our intended point, where we need our tangent.

⊛ Slope of tangent line at $x = \frac{\pi}{6}$

Need to find $f'\left(\frac{\pi}{6}\right)$.

$$f(x) = 6\sin x \Rightarrow f'(x) = [6\sin x]' = 6[\sin x]'$$
$$= 6\cos x.$$

So, $f'\left(\frac{\pi}{6}\right) = 6 \cdot \cos\frac{\pi}{6} = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$

⊛ point-slope form to get tangent.

Our tangent is a straight line passing through $\left(\frac{\pi}{6}, 3\right)$ with a slope $3\sqrt{3}$.

Then by the point-slope form we get:

$$(y-3) = 3\sqrt{3}\left(x - \frac{\pi}{6}\right)$$

$$\Rightarrow \boxed{y = 3\sqrt{3}\,x + \left(3 - \frac{\sqrt{3}\pi}{2}\right)}$$

② Find $\frac{d}{dt}\left[\frac{t\cos t}{(1+t)}\right]$

$$\frac{d}{dt}\left[\frac{t\cos t}{(1+t)}\right] = \frac{(1+t)\frac{d}{dt}[t\cos t] - t\cos t\,\frac{d}{dt}[(1+t)]}{(1+t)^2}$$

$$= \frac{(1+t)\left[\frac{d}{dt}(t)\cdot\cos t + t\cdot\frac{d}{dt}(\cos t)\right] - t\cos t\,(1)}{(1+t)^2}$$

$$= \frac{(1+t)\left[1\cdot\cos t + t(-\sin t)\right] - t\cos t}{(1+t)^2}$$

$$= \frac{(1+t)\cos t - t\sin t - t\cos t}{(1+t)^2}$$

$$= \frac{\cos t + \cancel{t\cos t} - t\sin t - \cancel{t\cos t}}{(1+t)^2}$$

$$= \frac{\cos t - t\sin t}{(1+t)^2}$$